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SOFTWARE FOR EVALUATION OF UNCERTAINTY IN MEASUREMENTS

Abstract: Software ASSISTANT enabling calculation of uncertainty of measurements is presented in this paper. Mathematical algorithm used by Assistant was developed according to recommendations of European Co-operation for Accreditation. Assistant is equipped with a user friendly interface and offers its users a series of examples of uncertainty calculations. Reports generated by Assistant can be saved, printed or exported to popular formats: Microsoft Word, Microsoft Excell or RTF. All these features make Assistant a useful tool in centres that implemented quality systems, in scientific laboratories or in teaching metrology.

Key words: *metrology, uncertainty, software, ISO 9000, ISO 17025*

1. INTRODUCTION

There is a general trend in industry to accept the quality systems according to the international norms ISO 900-9004, EN 45001-45003, ISO/IEC 17025. According to these documents the industrial plants and accreditation laboratories are required to use reliable measuring devices and to evaluate an uncertainty of measurements and present it in the certificates.

The well known "Guide to the expression of uncertainty in measurement", commonly termed GUM (or *GUIDE*), published in 1993 in the name of the seven main international metrological organizations recommends uncertainty as measure of measurement accuracy and presents rules of its calculations [1]. These rules enables calculation of combined standard uncertainty or expanded uncertainty of measure-

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ment result even in case of complex measuring systems. However, uncertainty calculations are time consuming even in case of simple measurement models. Additionally there is still too few publications in literature presenting exemplary uncertainty calculations. Therefore proper implementation of recommendations of the GUM is one of the main obstacles for further growth of quality systems in industry.

Software Assistant enabling easy and speedy calculations of uncertainty of measurement results is presented in this paper.

2. INTERNATIONAL SITUATION

The GUM has not found immediately wide acceptance all over the world. It is still a common opinion in many industrial communities that the GUM has too high requirements for middle-rank technical personnel and that creates unnecessary bureaucracy; or in part of scientific community that it represents too simplistic view on accuracy analysis.

However, in spite of the mentioned above reservations nowadays the GUM method of evaluating and expressing measurement uncertainty has been adopted widely by industry and commerce.

GUM methods have been also adopted by various regional metrology and related organizations including: European Cooperation for Accreditation, EUROLAB, European Collaboration in Measurement Standards, American Association for Laboratory Accreditation, National Voluntary Laboratory Accreditation Program, North American Collaboration in Measurement Standards

Moreover, the GUM has been adopted by almost all national metrology institutes throughout the world, such as the National Research Council (NRC) in Canada, the National Physical Laboratory (NPL) in the United Kingdom, and the Physikalisch-Technische Bundesanstalt in Germany. Most recently, the GUM has been adopted by the American National Standards Institute (ANSI) as an American National Standard.

In 1998 there was founded the International Accreditation Forum IAF as a part of Guangzhou Agreement – Chiny due to strong globalisation trend present in world economy. Importance of this organisation arise from the fact that its members who signed multilateral agreement respect certificates issued by any of them and this situation significantly supports international commerce.

European Cooperation for Accreditation plays the leading role in IAF in field of metrology. EA has published over a hundred publication that present precise examples of calculation of uncertainty in many measuring processes. EA publications are accepted as standards of uncertainty calculations by other international organisations.

3. ASSISTANT - MATHEMATICAL ALGORITHM

Mathematical algorithm used by ASSISTANT software is based on recommendation of the EA publication EA-4/02 „Expression of uncertainty of measurement in calibration", that is the main normative document regulating EA guidelines in field of uncertainty evaluation.

Combined standard uncertainty $u_c(y)$ is calculated using the formula (1) in case of non-correlated input quantities, or formula (2) in case of correlated input quantities:

$$u_c(y) = \sqrt{\sum_{i=1,n} c_i^2 u(x_i)^2} \tag{1}$$

$$u_c(y) = \sqrt{\sum_{i=1,n} c_i^2 u(x_i)^2 + \sum_{\substack{i,k=1,n \\ i \neq k}} c_i c_k u(x_i, x_k)} \tag{2}$$

where:

$$c_i = \frac{\partial f}{\partial x_i} = \left.\frac{\partial f}{\partial X_i}\right|_{X_1 = x_x, \ldots, X_N = x_n}, \quad u(x_i, x_k) = u(x_i)u(x_k)r_{ik}$$

$$r_{ik} = \frac{1}{n(n-1)} \sum_{j=1}^{n} (x_{ij} - \overline{x_i})(x_{kj} - \overline{x_k}),$$

c_i – sensitivity coefficient, r_{ik} – correlation coefficient.

Expanded uncertainty U of the measurand estimate y is calculated as

$$U = k \cdot u_c(y) \tag{3}$$

where k is the coverage factor.

ASSISTANT user can choose four methods of determination of the coverage factor k.

First, the user can set value of the coverage factor k from within the range from 1 to 4 (default value k=2) by himself.

Second, the user determines the required level of confidence and Assistant calculates k using the Welch-Satterthwaite formula

$$v_{eff} = \frac{u^4(y)}{\sum_{i=1}^{n} \frac{c_i^4 u_i^4(x)}{v_i}}, \tag{4}$$

where v_i is the degree of freedom of the input quantity *i*.

Next, degrees of freedom of input quantities are calculated using the formula (5) in case of quantities determined using A method. In case of quantities of normal distribution determined using B method Assistant proposes to use the formula (6), but in case of quantities of other distribution determined using B method – the formula (7)

$$v_i = n - 1, \tag{5}$$

$$v_i = 50, \tag{6}$$

$$v_i = \infty. \tag{7}$$

Formulas (5-7) mean that Assistant proposes to assume that the relative uncertainty of determination of the standard uncertainty u_i using B method is equal 0.1 in case of input quantity of normal distribution, or 0 in case of input quantity of other distribution. However, in case when there are serious doubts about validity of these assumptions the user can set its own values of the relative uncertainty; and indirectly he can sets own values of the degrees of freedom due to the relationship

$$v_i \approx \frac{1}{2}\left[\frac{\Delta u(x_i)}{u(x_i)}\right]^{-2}, \tag{8}$$

where the component in brackets is the relative uncertainty of determination of the standard uncertainty u_i.

Third method is used in case when an input quantity of rectangle distribution represents a dominant contribution to the combined standard uncertainty. The coverage factor is then calculated

$$k(p) = p\sqrt{3} \tag{9}$$

It is considered that the input quantity i=1 of rectangle distribution is the dominant quantity when the following condition is fulfilled

$$u_R(y)/u_1(y) < 0{,}3 \tag{10}$$

where $u_1(y)$ is the contribution of the dominant quantity i=1 to the combined standard uncertainty $u(y)$, $u_R(y)$ is the contribution of other quantities that can be calculated as

$$u_R = \sqrt{\sum_{i=2,n} u_i^2(y)}\ . \qquad (11)$$

Fourth method is used in case when two input quantities of rectangle distributions can be treated as dominant contribution to the combined standard uncertainty. The coverage factor is then calculated

$$k(p) = \frac{1}{\sqrt{\frac{1+\beta^2}{6}}} \cdot \begin{cases} \frac{p(1+\beta)}{2} \rightarrow \frac{p}{2-p} < \beta \\ 1-\sqrt{(1-p)(1-\beta^2)} \rightarrow \frac{p}{2-p} \geq \beta \end{cases} \qquad \beta = \frac{b}{a} = \frac{|a_1 - a_2|}{a_1 + a_2} \qquad (12)$$

where a_1, a_2 are half-width of two dominant rectangle distributions of the input quantities.

The dominance criterion is almost identical as in third method. The only difference is that, in formula (10) instead of $u_1(y)$ the contribution from the two dominant quantities $u_0(y)$ should be used, where

$$u_0(y) = \sqrt{u_1^2(y) + u_2^2(y)}\ . \qquad (13)$$

Application of the earlier described mathematical algorithm developed on the basis of EA recommendation creates situation when ASSISTANT generates identical calculation results as presented in all EA publications.

4. ASSISTANT – PRINCIPLE OF WORK

Graphical user interface of Assistant is based on five windows – if we insert required data then the current window hides and next window appears.

The user inserts project title, project description in the first window "Project Description".

The user insert mathematical model of uncertainty using Assistant equation editor in the second window "Mathematical model". He defines also all quantities included in the mathematical model.

The user gives available numerical data about the input quantities in the third window "Input data". He inserts here: observations in case of quantities determined using A method, distribution type and other parameter necessary to determine the standard deviation of quantities determined using B method.

Assistant presents results of calculations of correlation coefficients between quantities determined using the A method of the same observations in the fourth window "Correlation". The default value of the correlation coefficients between other input quantities is zero. However, the user can set by himself other values within the range from –1 to 1.

Fifth window "Results" is used for presentation of calculation results of the combined standard uncertainty and the expanded uncertainty of the output quantity Calculation results of the combined standard uncertainty are presented in form of uncertainty budget table. This tables presents a list of all input quantities, estimates of the input quantities, the standard uncertainties, types of probability distributions of these quantities, the sensitivity coefficients, and the uncertainty contributions of all the input quantities (Fig. 1).

Budget

Quantity	Estimate	Standard Uncertainty	Prob.Distribution	Sensitivity Coefficient	Uncertainty Contribution
R_s	10000.1	0.0025	Normal distribution	1	0.0025
δR_D	0.02	0.005774	Uniform distribution	1	0.005774
δR_TS	0	0.001588	Uniform distribution	1	0.001588
r_c	1	0	Triangular distributio	10000	0.004082
r	1	0	Observations	10000	0.000707
δR_TX	0	0.003175	Uniform distribution	-1	-0.003175

Output

Quantity	Type of Data	Estimate	Combined Standard Uncertainty
R_x	Result	10000.18	0.008328

Fig. 1. Uncertainty budget

Results of calculation of expanded uncertainty are presented in form of a table that gives information about estimate of the output quantity, the expanded uncertainty of this quantity, the coverage factor used to determine the expanded uncertainty, the combined standard uncertainty, and the confidence level (Fig. 2).

Coverage: Standard

Level of Confidence (p): 95.00 % Coverage Factor (k):

Update

Expanded Uncertainty

Result	Estimate	Expanded Uncertainty	Coverage	Standard Uncertainty	Confidence Lev
R_x	10000.18	0.016342	1.96	0.008328	95.00%

Fig. 2. Expanded uncertainty table

5. ASSISTANT – BASIC FEATURES

Most important features of the Assistant are presented below.

1. It is written in C++ Builder language and works in the following operational systems: Windows 95/98/ NT/ Millenium/2000.
2. Assistant is equipped with a user friendly multi-language interface
3. File Help offers detail information about the mathematical algorithm and the interface

1. It is possible to import of observations from a text files or through clipboard instead of manual inserting
2. Program generates reports in form recommended by EA
3. Reports can be saved, printed, or exported to such popular formats like Microsoft Word, Microsoft Excell, or RTF
4. It supports following probability distributions: normal, t-, U, rectangular, triangular, trapezoidal, cosine distribution, quadratic distribution
5. Assistant automatically calculates correlation between input quantities of the same number of observations determined using the A method
6. ASSISTANT generates identical calculation results as presented in all EA publications what confirm correctness of its mathematical algorithm.

The presented above features of Assistant creates from it a professional software that can be used for uncertainty evaluation.

6. ASSISTANT – COMPARING TO OTHER SOFTWARE

Uncertainty of measurement results can be calculated using a few well known commercially available software.

It is possible to use software „Statistica for quality" offered by StatSoft, that, in opinion of this company, is "a set of universal tools that can be used for quality control". This product of StatSoft represents powerful capabilities for statistical analysis of observation results. However, typical testing laboratory can use only a marginal fraction of these capabilities. What is more important, „Statistica for quality" does not offer any precise method for dealing with quantities determined using the B method, and does not generate reports according to EA guidelines.

Mathlab, Mathcad, Mathematica, Maple are well known commercially available software for symbolic and numerical calculations and graphical presentation of calculation results. It is probably possible to solve any uncertainty problems using these software. However, they do not have an integral uncertainty package and their user in institutions that implemented quality systems is required to write his own package. It is a serious disadvantage of these software due to necessity to learn languages used by these software.

Because of the above mentioned reasons direct application of these well known software for uncertainty calculation is possible but difficult. Assistant is a simple software but use a mathematical algorithm and user interface optimised for application in quality systems and does not have any of the above mentioned disadvantages.

5. CONCLUSIONS

The software presented in this paper – ASSISTANT – can be a useful tool for uncertainty evaluation, particularly in institutions that implemented quality systems according to international standards ISO 9000, EN 45000, ISO/IEC 17025.

There are two main advantages of the developed software ASSISTANT. First, it significantly reduces the time necessary to determine uncertainty of results of measurements. Second, due to user friendly interface and a set of examples it can teach its user methods of uncertainty calculation of many real measurement systems. Because of these significant advantages the software ASSISTANT can help to implement the recommendations of the GUM in many industrials plants and laboratories that so far do not estimate accuracy of measurement results due to problems connected with uncertainty calculations.

REFERENCES

[1] *Guide to the expression of uncertainty in measurement*, International Organisation for Standardisation-International Electrotechnical Commission-International Organisation of Legal Metrology-International Bureau of Weights and Measures, TAG 4/WG 3 (1993).

[2] Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, Technical Note 1297, 1994

[3] EA-4/02 Expression of the Uncertainty of the Measurement in Calibration, (1996- 1999)

[4] Publications EA 10-01÷EA 10-16 (1996-2001)